

# **Product Line Strategy under Complementary Effects**

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In this research, we study a durable goods manufacturer's product line strategy when the product interacts with those of a complementary industry. It is well known that the cannibalization between high- and low-end products can significantly lower durable goods manufacturers' total profits. This is why, in general, a durable goods manufacturer would not introduce a low-end product if the low-end product cannot be produced at a lower cost than the higher-end product. However, when the durable product is involved in a complementary relationship with another market and when such a connection is sufficiently strong, companies have an incentive to expand their product line, even when the low-end version costs the same as that of the high-end. A broader product line is a credible commitment to higher future output, and thus encourages higher output from the complementary industry, which, in turn, boosts the demand for the durable good.

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## **I. INTRODUCTION**

For companies whose products interact with those of a complementary industry, financial performance depends not only on the companies' own operating decisions but also on the decisions in the complementary industry. For example, the prerequisite for a high-definition television to reach the mass market is to have a sufficient amount of high-definition television programming available at affordable prices. Nevertheless, companies who provide high-definition broadcasting will increase the availability of high-definition programming only when they expect that high-definition television manufacturers can mass produce affordable television sets. Thus, television manufacturers may want to assure

high-definition programming providers that the future output of high-definition television sets will create enough demand for high-definition programming.

It is well known that the companies involved in a complementary relationship usually produce less than the optimal levels. If a firm that interacts with a complementary industry can commit to a higher output level in advance, then the complementary industry may increase the quantity in response to such a commitment, which, in turn, benefits the firm due to the complementary effects.

In this work, we focus on the signaling effects of a firm's product line strategy and argue that providing a broader product line can constitute a credible promise of a greater output level of the durable good to the

complementary industry. We contrast the strategic considerations of a durable goods manufacturer whose product is affected by a complementary industry with manufacturers whose products are independent of other markets. In general, the purpose of providing vertically differentiated products to diversified consumers is to pursue higher profits through discrimination when the lower-end version of the durable good can be made at a significantly lower cost. Our study shows, however, that product line decisions are also strategically important in encouraging outputs from the complementary industry.

The remainder of our paper is organized as follows: After reviewing the related literature in Section 2, we then introduce in Section 3 a model that captures the complementary effects between a durable goods manufacturer and a complementary industry. In Section 4, we present the product line decisions of the durable goods manufacturer when the durable product can be differentiated. Finally, in Section 5, we summarize the paper and propose future research.

## II. RELATED LITERATURE

Bulow et al. (1985) show that a firm's actions in one market can change competitors' strategies in a second market. We establish that a firm's decisions in one market can change the incentives of the producer of a complementary product. Complementary effects have long been a subject of interest in the economics literature. Katz and Shapiro (1985, 1994), Farrell and Saloner (1985, 1986), Choi (1994), and Liebowitz and Margolis (1994) discuss the strategic issues (e.g., R&D, competition) involved when two products are complementary to each other and are made by the same manufacturer. Recent work by Parker and VanAlstyne (2003a, 2003b) shows that, by subsidizing one of the complementary markets, a monopolist that sells to both markets may

gain significantly more profits. In this research, we study how one product's production line decisions affect the complementary industry when two complementary products are made by separate firms. Bhaskaran and Gilbert (2005) demonstrate that, when an independent complementary industry exists, a monopoly durable goods manufacturer is better off leasing the product. We study a similar business situation, but we examine the effects of product line decisions instead of leasing options.

Bhargava and Choudhary (2001) show that a monopolist provides only a single version of the product without consideration of complementary effects. We draw on the classic literature on market segmentation and product line strategies (Moorthy & Png, 1992; Mussa & Rosen, 1978) and establish that a monopolist that interacts with a complementary market chooses a broader product line than would be chosen if no complementary product were available.

## III. THE BASIC MODEL

We consider a durable good, Product A, and its non-durable complement, Product B. These two products are produced by two separate firms. Firm A can choose to offer two different versions of Product A with quality  $s_L < s_H = 1$  or provide a single version of the product to the market. It is assumed that there are  $M$  consumers who will buy one unit of A or none. In the absence of Product B, a consumer's utility derived from consuming a unit of Product A is  $s \cdot v_A$ , where  $s$  is the quality of the product and  $v_A$  is a random variable that is uniformly distributed over  $[a_A - M, a_A]$ , where  $0 \leq a_A \leq M$ .  $v_A$  is a measure of consumers' utility per "unit" of quality from consuming A and  $s_L \cdot v_A$ , and  $s_H \cdot v_A$  are consumers' reservation prices for low- and high-end versions of product A. The variable cost for producing a unit of A, regardless of quality, is assumed to be  $c_A$ . The constant

marginal costs across qualities is a plausible assumption if we consider the products that are digital in nature or have high margin markups.

For simplicity, the complementary Product B is assumed to be non-differentiated, and the variable production costs are normalized to zero. If a consumer does not have the use of Product A, his or her marginal utility for the  $y^{th}$  unit of Product B can be expressed as  $(a_B + \varphi v_A - y)/\gamma$ , where  $a_B, \gamma \geq 0$  and  $\varphi \in (0, 2\gamma)$  are constants. Note that, if  $\varphi$  is strictly positive, then consumers with the highest valuations for the use of A will have the highest marginal utilities for product B. If  $\varphi = 0$ , then all consumers are homogeneous with respect to their marginal utilities for the complement. If a consumer has the use of either a high-end or low-end version of Product A, then his or her marginal utility for the  $y^{th}$  unit of Product B increases by  $k/\gamma$ . Thus, the marginal utility for the  $y^{th}$  unit of Product B becomes  $(a_B + k + \varphi v_A - y)/\gamma$ . Here we assume the same  $k$  for both high-end and low-end versions of Product A. The assumption will simplify the analysis in this paper. Nevertheless, most results can still be achieved if we assume different  $k$  for different versions.

We first derive the indifferent consumers. A consumer with valuation  $v_A$  will buy the following number of Product B, given the price of Product B ( $p_B$ ).

$$y_i(p_B, v_A, \delta) = a_B + k\delta + \varphi v_A - \gamma p_B \quad (1)$$

where  $\delta$  is a  $\{0,1\}$  valued indicator. If  $\delta = 1$ , the consumer possesses a unit of either a high-end or low-end version of Product A. Otherwise, the consumer buys only Product B.

A consumer's total utility from consuming a unit of Product A with high quality and  $y_i(p_B, v_A, 1)$  units of B is

$$(s_H \cdot v_A - p_{AH}) + \int_0^{y_i(p_B, v_A, 1)} \frac{a_B + k + \varphi v_A - x}{\gamma} dx - y_i(p_B, v_A, 1)p_B \quad (2)$$

where  $p_{AH}$  is the unit price for the high-end version of Product A.

The total utility from consuming a unit of A of low quality and  $y_i(p_B, v_A, 1)$  units of B is

$$(s_L \cdot v_A - p_{AL}) + \int_0^{y_i(p_B, v_A, 1)} \frac{a_B + k + \varphi v_A - x}{\gamma} dx - y_i(p_B, v_A, 1)p_B \quad (3)$$

where  $p_{AL}$  is the unit price for the low-quality version of Product A.

By setting Equations (2) and (3) equal, we can derive our marginal consumer who is indifferent in regard to buying a unit of a high-quality versus a low-quality version of Product A.

Such a marginal consumer needs to have  $v_A = v_A^1$ , which satisfies the following equation:

$$v_A^1 = \frac{p_{AH} - p_{AL}}{s_H - s_L}$$

The total utility for a consumer who buys only Product B is

$$\int_0^{y_i(p_B, v_A, 0)} \frac{a_B + \varphi v_A - x}{\gamma} dx - y_i(p_B, v_A, 0)p_B \quad (4)$$

By setting Equation (4) equal to (3), we derive the second type of marginal consumer, who is indifferent in regard to buying a low-end version of Product A or not buying A at all. This second type of marginal consumer needs to have  $v_A = v_A^2$ , which satisfies the following equation:

$$v_A^2 = \frac{2\gamma(kp_B + p_{AL}) - k(2a_B + k)}{2(s_L\gamma + k\varphi)}$$

Based on the utilities of the marginal consumers, we can derive the inverse demand functions for both the high-end and low-end versions of Product A. Let  $Q_{AH}$  and  $Q_{AL}$  be the sales of the high- and low-end versions of Product A, respectively. We assume that

$$a_A < \frac{4\gamma(c_A + 2Ms_L) + k(-2a_B + 2k + 7M\varphi)}{4\gamma s_L + 2k\varphi}$$

so that we have  $Q_{AH} + Q_{AL} < M$ .

With this assumption, the inverse demand functions for the high- and low-end versions of Product A and for Product B are derived for further analyses:

$$p_{AH}(Q_{AH}, Q_{AL}, y) = \frac{(k^2 + kM\varphi)(M - 2(Q_{AH} + Q_{AL})) + 2ky + 2M\gamma(a_A - Q_{AH} - s_L Q_{AL})}{2M\gamma} \quad (5)$$

$$p_{AL}(Q_{AH}, Q_{AL}, y) = \frac{(k^2 + kM\varphi)(M - 2(Q_{AH} + Q_{AL})) + 2ky + 2M\gamma s_L(a_A - Q_{AH} - Q_{AL})}{2M\gamma} \quad (6)$$

$$p_B(Q_{AH}, Q_{AL}, y) = \frac{2Ma_B + 2k(Q_{AH} + Q_{AL}) - 2y + M\varphi(2a_A - M)}{2M\gamma} \quad (7)$$

where  $y$  is the number of units of Product B that are made available to consumers.

#### IV. PRODUCT-LINE DECISIONS

We assume that Firm A produces and sells one or two versions of Product A directly to the market. If both versions are introduced, then more consumers will buy the low-end product, which will certainly cannibalize the high-end product. Consequently, Firm A may

not choose to introduce both versions because the low-end product will certainly cannibalize the high-end product. If Product A has no complementary products with which to interact, the product line decisions are made to ensure the best trade-off between increased volume and the switching of high-end consumers to the low-end product. When Products A and B are complements and are produced by different companies, Producer B's reaction to Firm A's product line decisions should be taken into consideration when Firm A makes the product-line decision.

##### 4.1. When a Single Version is Provided

If Firm A offers only one version of Product A, i.e., a low-end and a high-end version, but offers only one to consumers, the firm would introduce the high-end product, as the production costs are the same for both the high-end and low-end versions, but consumers place more value on the high-end product. This can be verified by comparing Firm A's profits under a single product of high quality and a single product of low quality.

The inverse demand functions when only the high-end version of Product A is offered, as shown below:

$$p_A(Q_H, y) = \frac{(k^2 + kM\varphi)(M - 2Q_H) + 2ky + 2M\gamma(a_A - Q_H)}{2M\gamma} \quad (8)$$

$$p_B(Q_H, y) = \frac{2Ma_B + 2k(Q_H) - 2y + M\varphi(2a_A - M)}{2M\gamma} \quad (9)$$

Thus the profit functions of Firms A and B can be expressed as:

$$\pi_A^{SD}(Q_H, y) = Q_H(p_A(Q_H, y) - c_A) \quad (10)$$

$$\pi_B^{SD}(Q_H, y) = yp_B(Q_H, y)$$

$$(11) \quad \pi_A^{SDI*} = \frac{(a_A - c_A)^2}{4} \quad (13)$$

where  $p_A(Q_H, y)$  and  $p_B(Q_H, y)$  are from Equations (8) and (9).

In equilibrium, each firm determines its output to maximize its own profits. These equilibrium quantities can be identified by simultaneously solving the first-order conditions for (10) and (11) with respect to  $Q_H$  and  $y$ , respectively. The equilibrium solutions are provided below:

$$Q_H^* = \frac{M(2k(a_B + k) - 4\gamma c_A + kM\varphi + 2a_A(2\gamma + k\varphi))}{2(3k^2 + 4M\gamma + 4KM\varphi)}$$

$$y^* = \frac{M}{2(3k^2 + 4M\gamma + 4KM\varphi)} (k^3 + 2k\gamma(a_A - c_A) - M\varphi(k^2 + 2M\gamma + 2kM\varphi) + (4a_B + 2a_A\varphi)(k^2 + M\gamma + kM\varphi))$$

Substituting the resulting equilibrium output quantities back into Equations (10) and (11), we have:

$$\pi_A^{SD*} = \frac{M(k^2 + M\gamma + kM\varphi)(2k(a_B + k) - 4\gamma c_A + kM\varphi + 2a_A(2\gamma + k\varphi))^2}{4\gamma(3k^2 + 4M\gamma + 4KM\varphi)^2} \quad (12)$$

$$\pi_B^{SD*} = \frac{M}{4\gamma(3k^2 + 4M\gamma + 4KM\varphi)^2} (k^3 + 2k\gamma(a_A - c_A) - M\varphi(k^2 + 2M\gamma + 2kM\varphi) + (4a_B + 2a_A\varphi)(k^2 + M\gamma + kM\varphi))^2 \quad (13)$$

When  $k = 0$ , i.e., when there is no complementary interaction between Products A and B, Firm A's optimal output and profits are:

$$Q_H^{SDI*} = \frac{a_A - c_A}{2} \quad (12)$$

## 4.2. When Both Versions are Provided

If Firm A offers both high-end and low-end versions of product A, more consumers will be persuaded to own a unit of product A (either high-end or low-end). The profit functions of Firm A and Firm B are shown as:

$$\pi_A^{BD}(Q_{AH}, Q_{AL}, y) = Q_{AH}(p_{AH}(Q_{AH}, Q_{AL}, y) - c_A) + Q_{AL}(p_{AL}(Q_{AH}, Q_{AL}, y) - c_A) \quad (14)$$

$$\pi_B^{BD}(Q_{AH}, Q_{AL}, y) = yp_B(Q_{AH}, Q_{AL}, y) \quad (15)$$

where  $p_{AH}(Q_{AH}, Q_{AL}, y)$ ,  $p_{AL}(Q_{AH}, Q_{AL}, y)$ , and  $p_B(Q_{AH}, Q_{AL}, y)$  are defined in Equations (5), (6), and (7).

Firm A and Firm B set their own output to maximize their respective profits. By applying first-order conditions to Equations (14) and (15), we can determine the equilibrium output quantities of both Product B and high-end and low-end versions of Product A:

$$Q_{AH}^{BD*} = \frac{a_A}{2} \quad (16)$$

$$Q_{AL}^{BD*} = \frac{M(2k(a_B + k) - 4\gamma c_A + kM\varphi) - a_A k(3k + 2M\varphi)}{2(3k^2 + 4M\gamma_{SL} + 4KM\varphi)} \quad (17)$$

$$y^{BD*} = \frac{M}{2(3k^2 + 4M\gamma_{SL} + 4KM\varphi)} ((k^3 + 2k\gamma(a_{ASL} - c_A) + k^2\varphi(4a_A - M) + 2M\varphi(2a_A - M)(\gamma_{SL} + k\varphi) + 4a_B(k^2 + M\gamma_{SL} + kM\varphi))$$

By studying Equation (17), we notice that, when  $M(2k(a_B + k) - 4\gamma c_A + kM\varphi) - a_A k(3k + 2M\varphi) > 0$ , Firm A would provide the low-end version in addition to the high-end version. Otherwise, Firm A will have only a standard product that is of high quality.  $M(2k(a_B + k) - 4\gamma c_A + kM\varphi) - a_A k(3k + 2M\varphi)$  is continuous in  $k$  and negative when  $k = 0$ . It equals zero at some  $k_1 < 0$  and  $k_2 > 0$ . As  $k$  is positive by our assumption, it follows that only when the complementary effects are strong enough, i.e.,  $k > k_2 > 0$ , would Firm A offer both versions to the market. We denote this threshold  $k$  beyond which Firm A would provide both versions of Product A as  $K_D$ .

Note that it is straightforward to verify that, when  $k > K_D$ , the total output of the high-end and low-end products is larger than the output when only the high-end product is provided.

Substituting the resulting equilibrium output quantities back into Equations (14) and (15), we have firms' equilibrium profits  $\pi_A^{BD*}$  and  $\pi_B^{BD*}$  when  $k > K_D$ .

**Proposition 1.** *When  $k > K_D$ , the complementary effect between Durable Good A and Non-durable Good B is strong enough to persuade Firm A to expand its product line to include both the high-end and low-end versions of A. Otherwise, Firm A offers only the high-end version to the market.*

Proposition 1 states that, when the complementary effect is strong enough, Firm A would offer both versions to the market. By introducing a low-end version, the total number of customers who will buy Product A would increase and, thus, would encourage Firm B to increase the output of B as well. As a result, customers may be willing to pay more for Product A, as Products A and B are complements. When  $k > K_D$ , such a complementary network effect is strong, and Firm A would benefit from introducing the low-end version.

**Corollary 1.** *When Product A is independent of any other product, i.e.,  $k = 0$ , Firm A provides only the high-end version of Product A.*

Corollary 1 follows from the fact that, when  $k = 0$ , the equilibrium low-end quantity is negative. The corollary confirms that Firm A behaves differently when the firm's product is independent of other markets than when the product interacts with those of a complementary industry. Under complementarity, Firm A may provide a broader product line than it would if the firm's product is independent of other markets. The corollary also shows that the purpose of introducing the low-end version into the context is to assure the complementary product producer of higher output levels of Product A. A broader product line caters to more consumers and could be understood as a credible commitment to a higher output level. If firm B anticipates that more of Product A would be sold, the firm will increase its output level as well, which, in turn, will increase the consumers' willingness to pay for Product A. Our results show that the complementary effects can offset the cannibalization between high-end and low-end products and make a broader product line an attractive option for a durable goods manufacturer.

Note that Corollary 1 and the results of Bhargava and Choudhary's (2001) research are very similar but are derived through the use of different models. Their paper shows that, if we can cost effectively produce the high-end product relative to the low-end product, and the distribution of the customer satisfies the increasing failure rate property, then providing only the high-end product will bring the maximum profits to the company. Intuitively, because the high-end product may have a much higher profit margin, due to constant variable production costs across qualities, the loss from cannibalization is large and, thus,

introducing the low-end product is not an attractive choice.

**Proposition 2.** *For Firm A, interacting with a complementary Product B, the quality level of the low end has no effect on the firm's decision regarding whether to include the low end in the product line.*

Recall that, when  $M(2k(a_B + k) - 4\gamma c_A + kM\varphi) - a_A k(3k + 2M\varphi) > 0$ , Firm A would provide the low-end version in addition to the high-end version. Because  $M(2k(a_B + k) - 4\gamma c_A + kM\varphi) - a_A k(3k + 2M\varphi)$  does not contain  $s_L$ , Proposition 2 holds.

In reality, there may be multiple quality levels available for the low-end product, in which case the durable goods manufacturer's product line decision has nothing to do with the selected quality level for the low-end product.

**Proposition 3.** *When Firm A offers a broader product line that includes both versions of Product A, the firm's profit decreases with the quality of the low-end version  $s_L$ . Further, the firm's output of the low-end version of product A  $Q_{AL}^{BD*}$  also decreases with  $s_L$ .*

Proof:

Recall that Firm A offers both versions of Product A when  $k > K_D$ . Take the first-order derivative of  $\pi_A^{BD*}$  with respect to  $s_L$ . It can be shown that, under the condition  $k > K_D$ , the derivative is negative.

The second result is from Equation (17), which shows that the output of the low-end version of product A  $Q_{AL}^{BD*}$  decreases in  $s_L$ .

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The above proposition shows that, if both versions are provided, Firm A has an incentive to lower the quality of the low-end version to get more consumers to buy. Doing so will further differentiate the high-end and

low-end products and increase Firm A's net gains.

## V. CONCLUSION

In this research, we studied a durable goods manufacturer's product-line strategy when the manufacturer's product interacts with those of a complementary industry. Although a broader product line can serve as an assurance to the complementary industry that there will be a higher output level of the durable good, such a product line creates cannibalization of the high-end product by the low-end product. Under the extreme situation in which the production costs are the same for both versions, and the high-end product enjoys a much higher margin, the cannibalization can present a major threat to a company's effort to maximize the profits. This is why, without sufficiently strong complementary effects, a durable goods manufacturer would not introduce a low-end product if the low-end product cannot be produced at a lower cost.

When the durable good is involved in a complementary relationship with another market and when such connection is sufficiently strong, however, the durable goods manufacturer has an incentive to expand the product line. A broader product line is a credible commitment to higher future output and, thus, encourages higher output from the complementary industry, which, in turn, would boost the demand for the durable product.

The quality of the low-end product has no effect on a durable goods manufacturer's product-line decision. Further, when the low-end version of the durable good is offered, under the condition of complementary effects, a durable goods manufacturer prefers to set the quality of the low-end product at the lowest possible level to maximize the manufacturer's profits.

Our model assumes constant variable costs across qualities, but different qualities may be produced at different costs. Thus, it

would be worthwhile to determine the extent to which firms can extend their product line based on different cost-quality functions. It also would be valuable to conduct additional empirical studies on the output and product-line decisions of companies that interact with a complementary industry.

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